Lesson 2: Polygons

In this lesson, you will review convex, concave, and irregular polygons. You will determine how to find the area and perimeter of a polygon, as well as the circumference of a circle. You will examine how changes in a polygon's dimensions affect its area and perimeter. You will use properties of congruent and similar polygons to solve problems. Finally, you will create and confirm tessellations using polygons.

Polygons

A polygon is a closed 2-dimensional figure made up of line segments. A **regular polygon** is a polygon where all the sides are congruent. A **convex polygon** is a polygon where every interior angle is less than 180°. The following table shows examples of regular convex polygons.

Polygon	Name	Number of Sides and Angles
	triangle	3
	quadrilateral	4
\bigcirc	pentagon	5
	hexagon	6
	octagon	8
	decagon	10

An **irregular polygon** is a polygon with sides of different lengths. A **concave polygon** is a polygon where at least one angle has a measure greater than 180°. The following table shows different examples of concave polygons. All concave polygons are also irregular polygons.

Quadrilateral	Pentagon	Hexagon	Octagon	Decagon
				λ

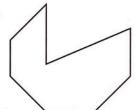
) Practice

Directions: For questions 1 through 4, label the polygon based on its number of sides and identify whether it is concave or convex and regular or irregular.

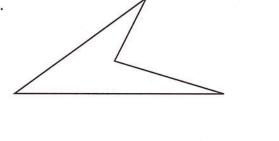
1.



3.



2.



4.



Directions: For questions 5 through 8, draw the polygon based on its description.

5. regular convex pentagon

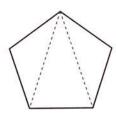
7. irregular concave quadrilateral

6. regular convex octagon

8. irregular concave hexagon

Interior and Exterior Angles

An interior angle of a polygon is formed by two adjacent sides of a polygon. You can find the sum of the measures of the interior angles of any polygon just by knowing the number of sides it has. Any polygon can be divided into triangles by drawing diagonals from one vertex to each of the remaining vertices.



The number of triangles formed in this way is always two less than the number of sides of the polygon. The sum of the measures of the interior angles of a triangle is 180°. The sum of the measures of the interior angles of any polygon is given by the following formula:

sum of interior angles =
$$180^{\circ}(n-2)$$

where
$$n =$$
 number of sides of the polygon

Because a regular polygon has congruent side lengths, its interior angles will also have congruent measures. Therefore, you can divide the sum of a polygon's interior angles by the number of interior angles to find the measure of any interior angle. You can use the following formula for a regular polygon:

measure of any interior angle =
$$\frac{180^{\circ}(n-2)}{n}$$
 where $n=$ the number of sides of

where
$$n =$$
 the number of sides of the regular polygon



Find the sum of the measures of the interior angles of a hexagon.

sum of interior angles =
$$180^{\circ}(n-2)$$

= $180^{\circ}(6-2)$
= $180^{\circ}(4) = 720^{\circ}$

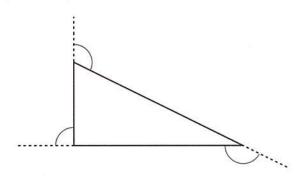
The sum of the measures of the interior angles of a hexagon is 720°.



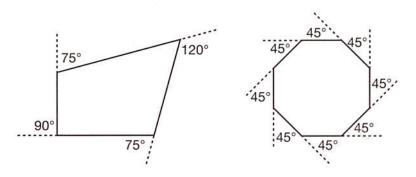
TIP: When given the sum of the measures of the interior angles of a polygon, you can use the same formula to find the number of sides the polygon has.

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An exterior angle of a polygon is formed by one side of a polygon and the extension of its adjacent side outside the polygon. The figure below shows a triangle with its exterior angles labeled.



The sum of the exterior angles of a polygon is always 360°. It does not matter how many sides the polygon has or whether it is convex, concave, regular, or irregular.



The irregular quadrilateral has four exterior angles with measures of 75° , 75° , 90° , and 120° . Their sum is 360. The regular octagon has eight exterior angles with measures of 45° each. Their sum is 360° as well.

Because a regular polygon has interior angles with congruent measures, its exterior angles will also have congruent measures. Therefore, you can divide the sum of a polygon's exterior angles, 360°, by its number of sides to find the measure of each exterior angle. You can use the following formula for a regular polygon:

measure of any exterior angle =
$$\frac{360^{\circ}}{n}$$
 where $n =$ the number of sides of the polygon

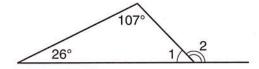
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Practice

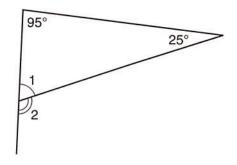
- 1. What is the sum of the interior angles of an octagon? _____
- 2. What is the measure of each exterior angle of a pentagon that has 5 congruent sides?
- 3. The sum of the interior angles of a polygon is 1620°. How many sides does the polygon have?
- 4. What is the sum of the exterior angles of the following polygon? _____



5. What is the measure of $\angle 2$?



6. What is the measure of $\angle 2$?

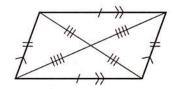


Quadrilaterals

A quadrilateral is a polygon with four sides. The sum of the measures of the interior angles of a quadrilateral is 360° . The following list shows different types of quadrilaterals.

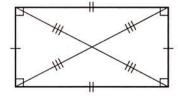
Parallelogram

A **parallelogram** has opposite sides that are parallel and congruent. The opposite angles of a parallelogram are congruent. The diagonals bisect each other. All adjacent interior angles are supplementary.



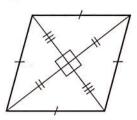
Rectangle

A **rectangle** is a parallelogram with four congruent, right angles. The diagonals of a rectangle are congruent. The diagonals bisect each other.



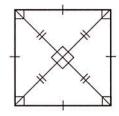
Rhombus

A **rhombus** is a parallelogram with four congruent sides. The diagonals of a rhombus bisect the interior angles and each other. The diagonals are perpendicular to each other.



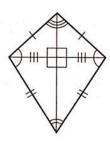
Square

A **square** is a parallelogram with four congruent sides and four congruent, right angles. The diagonals of a square are congruent. The diagonals also bisect the interior angles and each other. The diagonals are perpendicular to each other.



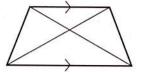
Kite

A **kite** is a quadrilateral with two pairs of adjacent, congruent sides. One pair of opposite angles is congruent. The diagonals of a kite are perpendicular to each other. One diagonal bisects the other. One diagonal bisects two interior angles.



Trapezoid

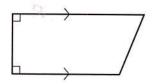
A **trapezoid** has exactly one pair of parallel sides. An isosceles trapezoid is a trapezoid with congruent base angles and congruent side lengths. A lower base angle of an isosceles trapezoid is supplementary to an upper base angle. The diagonals of an isosceles trapezoid are congruent.



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Example

What kind of quadrilateral is shown?

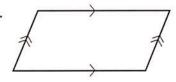


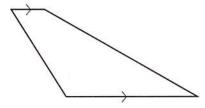
The figure shown has exactly one pair of parallel sides. A quadrilateral with exactly one pair of parallel sides is a trapezoid. Even though it may look different from the example in the table above, the figure is a trapezoid.

Practice

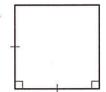
Directions: For questions 1 through 8, classify the quadrilaterals according to their sides, angles, and diagonals. Use as many of the following words that describe the figure: square, rectangle, rhombus, parallelogram, trapezoid, and kite.

1.

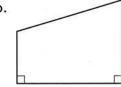




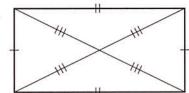
2.



6.



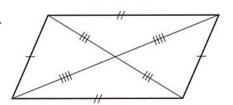
3.



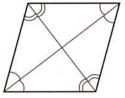
7.



4.

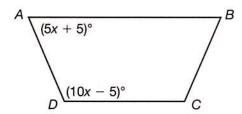


8.

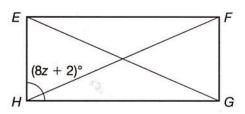


Directions: For questions 9 through 12, find the angle measures or variable values of the given quadrilaterals.

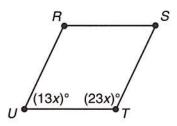
9. What is the measure of $\angle B$ in the isosceles trapezoid below?



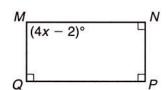
10. The diagonals in quadrilateral *EFGH* are congruent. The measure of $\angle GHE$ is $(8z + 2)^{\circ}$. What is the value of z?



11. What is the measure of $\angle R$ in the rhombus below?



12. What is the value of x in the quadrilateral below, in degrees?



Perimeter and Circumference

Perimeter (P) is the distance around the outside of a two-dimensional figure. This distance is called the **circumference (C)** when the two-dimensional figure is a circle. The following table shows formulas for finding the perimeter or circumference of some two-dimensional figures.

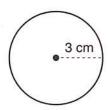
Figure	Formula	
Polygon s_1 s_2 s_3		where $s_i = \text{length of side } i$ des) $n = \text{number of sides}$
Regular Polygon	P=l • number of sides	where $l=$ length of each side
Rectangle w l	P = 2l + 2w or $P = 2(l + w)$	where $l = length$ $w = width$
Circle	$C = \pi d$ or $C = 2\pi r$	where $d =$ diameter $r =$ radius $\pi \approx 3.14$

TIP: The numbers lower than the letters are called **subscripts**. They are used to show the different sides of polygons: s_1 means side 1, s_2 means side 2, and so on.



Example

What is the circumference of the following circle?



Since the radius of the circle is given, use $C = 2\pi r$.

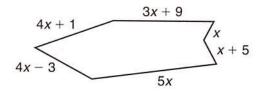
$$C = 2\pi r$$
$$= 2 \cdot \pi \cdot 3$$
$$= 6\pi$$

The circumference of the circle is 6π centimeters.



Example

The perimeter of the following figure is 138. What is the value of x?



The sum of the six sides on this hexagon equals 138. Set up an equation and solve for x.

$$138 = (4x + 1) + (3x + 9) + x + (x + 5) + 5x + (4x - 3)$$

$$138 = 18x + 12$$

$$126 = 18x$$

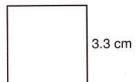
$$7 = x$$

The value of x is 7.

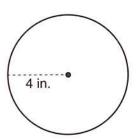
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Practice

1. What is the perimeter of the following square?



2. What is the circumference of the following circle?



- 3. What is the perimeter of an equilateral triangle with a side length of 20 cm?
 - A. 200 cm
 - B. 100 cm
 - C. 80 cm
 - D. 60 cm
- 4. If a regular pentagon (5-sided polygon) has a side length of x + 2, what is the perimeter of the pentagon?

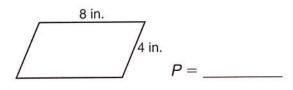
A.
$$x^2 + 4x + 4$$

B.
$$10x + 20$$

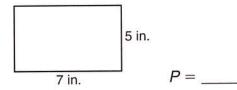
C.
$$5x + 10$$

D.
$$x + 7$$

5. What is the perimeter of the following parallelogram?



6. What is the perimeter of the following rectangle?



- 7. What is the perimeter of a regular octagon with a side length of 18 in.?
 - A. 144 in.
 - B. 128 in.
 - C. 112 in.
 - D. 96 in.
- 8. What is the circumference of a circle with a diameter of 10 m?
 - A. 5π m
 - B. 10π m
 - C. 20π m
 - D. 100π m

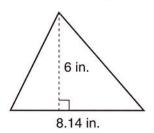
Area

Area (A) is the measure of the region inside a two-dimensional figure. Area is measured in square units. The following table shows formulas for finding the area of some two-dimensional figures.

Figure		Formula
Triangle	$A=\frac{1}{2}bh$	where $b = \text{base length}$ $h = \text{height}$
Square s s	$A = s^2$	where $s = $ length of each side
Rectangle w l	A = lw	where $l = length$ w = width
Parallelogram h	A = bh	where $b = \text{base length}$ $h = \text{height}$
Trapezoid $h = \begin{bmatrix} b_1 \\ b_2 \end{bmatrix}$	$A=\frac{1}{2}h(b_1+b_2)$	where b_1 = base 1 length b_2 = base 2 length h = height
Circle	$A = \pi r^2$	where $r = \text{radius}$ $\pi \approx 3.14$

Example

What is the area of the following triangle?



Substitute the values into the formula and solve.

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \cdot 8.14 \cdot 6$$

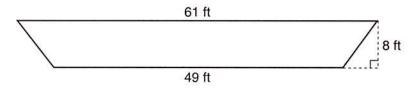
$$= 24.42$$

The area of the triangle is 24.42 in.²



Example

The Morelli family is carpeting a section of their hallway. The section of hallway is shown below. How many square feet of carpet will they need to cover this section of the hallway?



This section of the hallway is in the shape of a trapezoid. Substitute the values into the formula for the area of a trapezoid and solve.

$$A = \frac{1}{2}h(b_1 + b_2)$$

$$= \frac{1}{2} \cdot 8 \cdot (49 + 61)$$

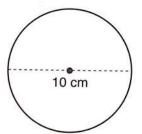
$$= 440$$

The Morelli family needs 440 ft² of carpet to cover the section of their hallway.



Practice

1. What is the area of a rug with the following dimensions?



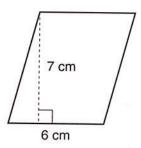
A =

2. What is the area of a square pizza with a side length of 12 in.?

A = ____

- 3. What is the area of a trapezoid with base lengths of 5 cm and 13 cm and a height of 8 cm?
 - A. 54 cm²
 - B. 64 cm²
 - C. 72 cm²
 - D. 81 cm²
- 4. What is the height of a triangle with an area of 30 m² and a base length of 6 m?
 - F. 5 m
 - G. 10 m
 - H. 24 m
 - I. 90 m

5. What is the area of the following parallelogram?



A =

6. The length of the rectangle below is twice the width (w). What is the area of the rectangle expressed in terms of w?

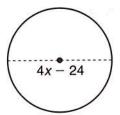
9 ===	1
- 1 =	w
	A =

A =

- 7. If a parallelogram has an area of 180 ft² and a height of 12 ft, what is the length of its base?
 - A. 5 ft
 - B. 15 ft
 - C. 24 ft
 - D. 30 ft

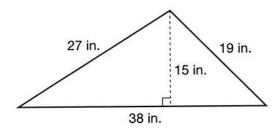
Benchmark Codes: MA.912.G.2.5

8. What is the area of the circle shown below?



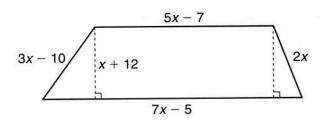
A = _____

9. Ginger is moving and needs to make sure she has enough room to fit her coffee table in the moving truck. The top of the coffee table is the shape of a triangle. Ginger is placing the table upside down inside the truck. The dimensions of the table top are shown below. How many square inches will the coffee table take up on the floor of the truck?



A = _____

10. What is the area of the figure shown below?



A = _____

11. Rosario is painting the floor of the pool in the backyard. The rectangular pool is 33 yards long, 15 yards wide, and 3 yards deep. How many square yards will Rosario have to paint?

A =

Changes in Dimensions

When the dimensions of a plane figure or solid figure change, you can quickly figure out how the perimeter, area, circumference, or volume changes.

Changes to Perimeter and Area

When **all** the sides of a polygon are multiplied by some factor n,

- the new perimeter is *n* times the original perimeter
- the new area is n^2 times the original area.

Example

Elise draws a 3-4-5 right triangle with a perimeter of 12 in. and an area of 6 in.² She then draws a similar triangle 5 times as large (n = 5). What are the perimeter and area of the larger triangle?

$$P_{\text{(original)}} \cdot n$$
 $A_{\text{(original)}} \cdot n^2$ $6 \cdot 5^2 = 150 \text{ in.}^2$

The perimeter of the new triangle is 60 in.; its area is 150 in.²

Changes to Circumference and Area

When the radius or diameter of a circle is multiplied by some factor n,

- the new circumference is *n* times the original circumference
- the new area is n^2 times the original area.

Example

Abe draws a circle with a radius of 2 cm, a circumference of 12.56 cm, and an area of 12.56 cm². He then draws a circle that is 6 times as large (n = 6). What are the circumference and area of the new circle?

$$C_{\text{(original)}} \circ n$$
 $A_{\text{(original)}} \circ n^2$ $12.56 \circ 6 = 75.36 \text{ cm}$ $12.56 \circ 6^2 = 452.16 \text{ cm}^2$

The circumference of the new circle is 75.36 cm; its area is 452.16 cm².

Note: These rules only work when *all* dimensions of a plane figure are changed by the same factor n.

Practice

1. Mrs. Hansen received a rectangular map of Florida for her classroom with a perimeter of 10 ft. She actually ordered a map that was twice as long and twice as wide as the one she received. What is the perimeter of the map that she ordered?

P = _____

2. Samantha drew a square with a perimeter of 32 cm. If she multiplies the length of the square by 4 and keeps the width the same, what will be the perimeter of her new polygon?

P = _____

3. A square has a perimeter of 16 ft and an area of 16 ft². What will be the perimeter and area if its dimensions are multiplied by a factor of 7?

P = _____

A = ____

4. A sail on a model sailboat in the shape of a right triangle has an area of 15 cm². If the height of the sail is increased to three times its original height, while the base of the sail stays the same, what will be the area of the new sail?

A = _____

Directions: Use the following information to answer questions 5 and 6.

From the roof, Mr. Sanchez took an aerial picture of his rectangular patio. In the picture, the patio has a perimeter of 10.5 in. and an area of 6.5 in.² The length and width of the actual patio are 36 times the length and width of the picture.

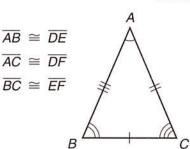
- 5. What is the perimeter of the actual patio?
 - A. 10 ft 5 in.
 - B. 23 ft 4 in.
 - C. 31 ft 6 in.
 - D. 36 ft 0 in.

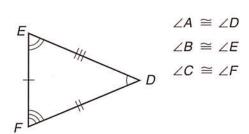
- 6. What is the area of the actual patio?
 - A. 7,431 in.2
 - B. 8,424 in.²
 - C. 16,500 in.²
 - D. 21,347 in.²

Congruent Polygons

Congruent polygons have the same shape and the same size. All corresponding segments and angles are congruent. The symbol for congruent is \cong .

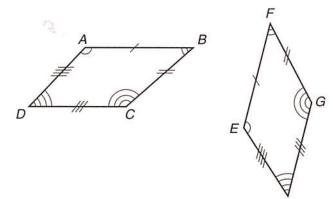
Given: $\triangle ABC \cong \triangle DEF$







Given: $ABCD \cong EFGH$, $m \angle A = 110^{\circ}$, $m \angle D = 60^{\circ}$, $m \angle G = 115^{\circ}$. What is the measure of $\angle F$?



Since the two figures are congruent $\angle A \cong \angle E$, $\angle B \cong \angle F$, $\angle C \cong \angle G$ and $\angle D \cong \angle H$. This means that $m \angle E = 110^\circ$ and $m \angle H = 60^\circ$. Since the interior angle sum of a quadrilateral is 360° we can solve for $m \angle F$.

$$360^{\circ} = m \angle E + m \angle F + m \angle G + m \angle H$$

 $360^{\circ} = 110^{\circ} + m \angle F + 115^{\circ} + 60^{\circ}$
 $360^{\circ} = 285^{\circ} + m \angle F$

$$75^{\circ} = m \angle F$$

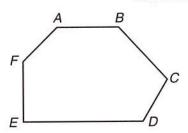
The measure of $\angle F$ is 75°.

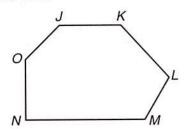
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Practice

Directions: Use the following figures to answer questions 1 through 5.

Given: ABCDEF ≅ JKLMNO





1. Which angle in polygon ABCDEF is congruent to ∠M?_____

4. Which pair of sides is congruent?

A.
$$\overline{AB}$$
 and \overline{NO}

B.
$$\overline{CD}$$
 and \overline{LM}

C.
$$\overline{FA}$$
 and \overline{AB}

D.
$$\overline{OJ}$$
 and \overline{CD}

5. Which pair of angles is congruent?

A.
$$\angle A$$
 and $\angle J$

B.
$$\angle B$$
 and $\angle D$

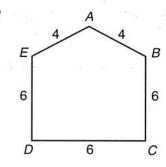
C.
$$\angle C$$
 and $\angle O$

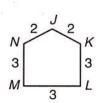
D.
$$\angle F$$
 and $\angle L$

Similar Polygons

Similar polygons have the same shape but not necessarily the same size. The symbol for similar is ~. The corresponding angles of similar figures are congruent, and the lengths of their corresponding sides are proportional. That means the lengths of any corresponding sides will have the same ratio. This ratio is also called the scale factor. In the following figures, the scale factor of ABCDE to JKLMN is 2:1. Each side of ABCDE is twice the length of its corresponding side in JKLMN.

Given: ABCDE ~ JKLMN





Corresponding angles:

$$\angle A \cong \angle J$$
 $\angle B \cong \angle K$ $\angle C \cong \angle L$ $\angle D \cong \angle M$ $\angle E \cong \angle N$

$$\angle C \cong \angle L$$

$$\angle D \cong \angle M$$

$$\angle E \cong \angle \Lambda$$

Corresponding sides:

$$\frac{AB}{JK} = \frac{4}{2} = 2$$

$$\frac{AB}{JK} = \frac{4}{2} = 2$$
 $\frac{BC}{KL} = \frac{6}{3} = 2$ $\frac{CD}{LM} = \frac{6}{3} = 2$ $\frac{DE}{MN} = \frac{6}{3} = 2$ $\frac{EA}{NJ} = \frac{4}{2} = 2$

$$\frac{CD}{LM} = \frac{6}{3} = 2$$

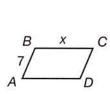
$$\frac{DE}{MN} = \frac{6}{3} = 2$$

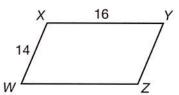
$$\frac{EA}{NJ} = \frac{4}{2} = 2$$



Example

Given: $ABCD \sim WXYZ$. What is the value of x?





Set up a proportion, multiply to get cross products, and solve the equation.

$$\frac{14}{7} = \frac{16}{x}$$

$$14 \cdot x = 7 \cdot 16$$

$$14x = 112$$

$$x = 8$$

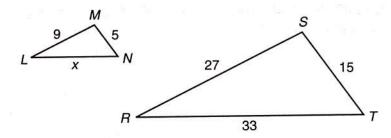
The value of x is 8.

0

Practice

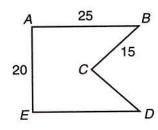
Directions: For questions 1 through 3, find the missing values.

1. Given: $\triangle LMN \sim \triangle RST$



x = _____

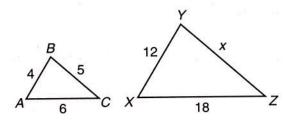
2. Given: ABCDE ~ FGHIJ





x = _____

3. Given: $\triangle ABC \sim \triangle XYZ$



x = _____

Tessellations

A **tessellation** is a group of congruent figures that covers a plane without gaps or overlaps.

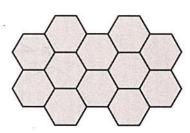
Because there can be no gaps or overlaps at each vertex in a tessellation, the polygons in a tessellation must fill the plane completely. The measures of all the interior angles where two or more polygons meet in a tessellation should sum to 360°. For a regular polygon to make a tessellation, the measure of each interior angle must be a divisor of 360°.



Example

The pattern is a tessellation that uses regular hexagons.

The interior angles of each regular hexagon in the pattern are 120°, which divides 360° evenly. The hexagons make a tessellation.

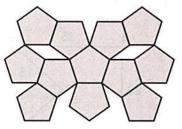




Example

The pattern uses regular pentagons. Do the pentagons make a tessellation?

The interior angles of each pentagon in the pattern are 108°, which is not a divisor of 360°. No, these pentagons do not make a tessellation.

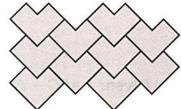




Example

The following pattern is a tessellation that uses irregular hexagons.

Each hexagon has five right angles and one 270° interior angle. In this tessellation, the hexagons meet in two ways: a 90° angle meets a 270° angle or two 90° angles meet a straight (180°) angle. In both cases the angle measures sum to 360°. The irregular hexagons make a tessellation.





TIP: There are only three regular polygons that will tessellate: triangles, squares, and hexagons. Any type of triangle can tessellate.

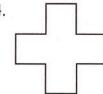
) Practice

Directions: For questions 1 through 6, identify whether the figure can be tessellated. If it can, draw six of the shapes tessellated.

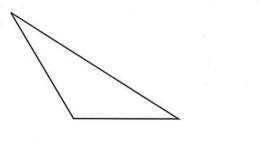
1.



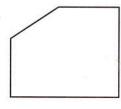
4.



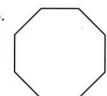
2.



5.



3.



6.

